The bio–inspired rain rule for Twenty20 cricket

The need

The youngest entrant to the cricketing family, T20 cricket, has been rising in popularity since its inception over a decade back. With several domestic T20 leagues and international T20s round the year, it is inevitable that weather plays a spoilsport. The rain rule has been the player's enigma and the spectator's frustration for long with the D/L rule coming under a lot of criticism and recently evolving into a seemingly improved version in DLS. The DLS, adopted by the ICC, uses a proprietary software to arrive at the target and is perceived by many players and fans as being technical mumbo–jumbo. The VJD rule, its closest competitor which has found limited support, is again computer–based but thankfully open–source and does not enjoy a significant superiority over the DLS for T20s. What if there were to be a rain rule for T20 cricket that is spectator–friendly – simple to understand, easy and inexpensive to use while giving accurate and fair revised targets for interrupted games ? The present article is a first step in this direction and we describe the evolution of a biologically motivated approach for the T20 rain rule.

The motivation

To devise a simple and accurate rain rule for T20 cricket, we turn to an interesting and unusual ally: Metabolism. Metabolism in animals, including humans, provide them with energy required for their sustenance and this complex phenomenon is governed by a simple three-quarter power–law relationship. Proposed in 1934 by Max Kleiber, the relationship maybe mathematically expressed as $B \sim M^{\frac{3}{4}}$, where B is the rate of metabolism and M is the mass of the animal. We hypothesize that the run-scoring ability of a team is analogous to the metabolism rate in animals and that the "mass" of a team can be defined in terms of the overs and wickets remaining during any stage of an innings. Consequently, we define the resource percentage of a team at any stage during an innings as $B = [f(O)g(W)]^{\frac{3}{4}}$, where O and W are overs remaining and wickets lost by the team at that stage. The functions f and g must be such that the resources decrease monotonically as O decreases and W increases and the specific choice of their parametric forms are given by,

$$f(O) = \frac{O}{20}$$

$$g(W) = \quad 1 - \exp\Big(\frac{W-10}{W+1}\Big)$$

This results in a simple expression for a team's resources (a value between 0% and 100% by construction) at any stage that reads,

$$B = \Big[\Big(\frac{O}{20}\Big)\Big(1 - \exp\Big(\frac{W-10}{W+1}\Big)\Big)\Big]^{\frac{3}{4}}$$

and is the basis for the target resetting methodology explained next.

The methodology*

The determination of the revised targets requires the calculation of the effective resources for each team, which is the resource percentage that is utilised or available for utilisation during each innings. The calculation of the effective resources can be done quite easily by the following step–by–step process.

1. Find the resources available at the start of each innings corresponding to the number of overs remaining at the start and W=0. This is 100% if the innings starts with all 20 overs available, but becomes lesser if there is a reduction of overs at the start, as can be seen from the expression for B.

2. For interruptions during the innings, determine the resources at the start and end of each interruption. This will correspond to the number of wickets lost at the interruption and the overs remaining both before and after the interruption. The difference between these resources at the start and end of the interruption is the resources lost due to each interruption. Adding up all such resources lost gives the total resources lost in the innings.

3. The effective resources for the innings is the difference between the resources at start and the total resources lost.

If S runs are scored in the first innings by utilising B_1 resources, then the projected first innings score is defined as $PS = \frac{S}{B_1}$. If the first innings was completed without interruptions (completion of 20 overs or the loss of

*Readers not desirous of understanding the logic leading to the target determination may choose to skip this part

10 wickets), $B_1 = 1$ and $PS = S$. However, for premature termination of the first innings due to rain interruptions, $B_1 < 1$ and the projected score is a *prediction* of what might have been the 20–over score given the situation at termination. If the effective resources for the second innings is B_2, then the par-score (to tie the match) is $P = PS \times B_2$ and the target (to win the match) is simply $T = P + 1$, both rounded off to the nearest integer. Since the resource calculation is key to deciding the target and is based on Kleiber's mathematical rule, we christen the target so obtained as the **"Kleiber target"** and the overall methodology as the Kleiber rain rule.

As a demonstrative example of the Kleiber rain rule, we consider the T20 game between Red Steel and Patriots in CPL 2015. The match started as a 20–over game, with Red Steel batting first. Rain stopped play when they were 56/1 after 9 overs and the match reduced to 14 overs per side. Red Steel finished at 134/5 in 14 overs. We need to find the 14–over Kleiber target for the Patriots, which is quite easy as shown below.

Resources at start for Red Steel = 1 (since O=20 and W=0)
Resources at start of first interruption = 0.6333 (since O=11 and W=1)
Resources at end of first interruption = 0.3506 (since O=5 and W=1)
Resources lost = 0.6333 – 0.3506 = 0.2827 (Only one interruption, hence this is the total resources lost)
Effective resources for Red Steel (B_1) = 1 – 0.2827 = 0.7173
Projected Score for Red Steel (PS) = 134/0.7173 = 186.82 runs
Resources at start for Patriots = 0.7653 (since O=14 and W=0)
Effective resources for Patriots (B_2) = 0.7653 – 0 = 0.7653 (No second innings interruptions, so no resources are lost)
Kleiber target = 186.82×0.7653 + 1 = 143.97 (rounded off as 144 runs)

We see that the Kleiber target is 144 runs, not very different from the DLS target of 142 runs for this case. In fact, for several scenarios, the difference of targets decided by these two rules is within 3 runs and the Kleiber target is then a "close approximation" to the DLS target.

The question of applicability

The applicability of any rain rule for T20 cricket primarily depends on its ability to satisfactorily describe the progression of a T20 innings. We put the Kleiber rain rule to this test by collating data from over 220 T20Is in the last five years. The data of fully completed first–innings were extracted from Cricinfo, and gave an average first-innings score of 156/6. We used the mathematical model to estimate the runs scored at the end of 5, 10 and 15 overs of the first innings (factoring the wickets lost at each of these stages) and compared them with actual observed scores. This is depicted in the first three panels of Figure 1, that indicate reasonable agreement between the model-estimates and observed data. This fair agreement is also observed between the estimated and observed runs scored in each of the four equal over-groups that the first innings has been divided into, as shown in the last panel of Figure 1. These studies are evidence for the potential of the Kleiber rain rule to model a typical T20 innings and gives confidence to apply it to practical scenarios.

The consistency and reasonability of Kleiber target

Any good rain rule must satisfy a set of seven objectives laid down by the ICC, and two of the most important ones are the consistency and reasonability of the revised targets. Indeed, much of the debates between DLS and VJD in the past have focussed on these two objectives. We take a detailed look at these objectives, specific to T20s, through three different scenarios as discussed below.

Scenario 1: 20–over first innings, 10–over second innings

This is a typical scenario, pertinent to T20s, discussed in articles by Rajesh and Stern. Figure 2 shows the observed final scores (for the same data set used earlier) and its relation with the 10–over score, categorized by wickets lost in four panels. The solid line is the model–estimated first–innings total, which has been projected based on the observed 10–over score. It can be seen that the underlying mathematical model is quite accurate, with the deviations in the last panel (for W=4) likely due to the lesser number of data points. Table 1 shows the 10–over Kleiber and DLS targets as well as the 10–over Kleiber par-score (for no wickets lost). The sum of the 10–over Kleiber par-score (for no wickets lost) and 10–over Kleiber target is always equal to the target for an uninterrupted match which a necessary consistency criterion that is satisfied by DLS but not by VJD. Furthermore, the 10–over Kleiber and DLS targets are quite close, and start to differ only when the first innings totals become higher.

Scenario 2: 10–over first innings, 5–over second innings

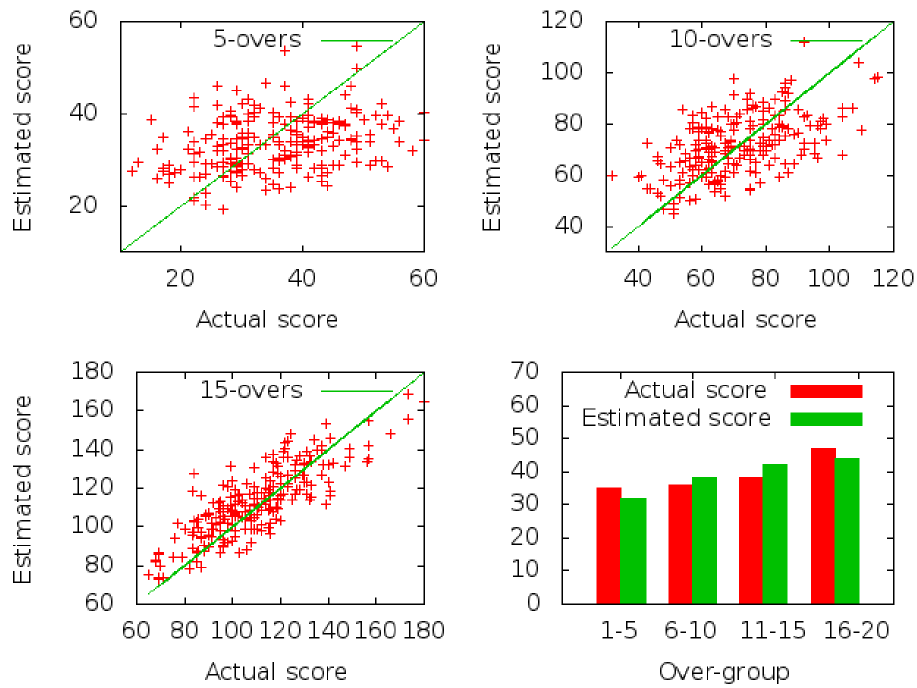

Figure 1: Panels 1-3: Comparison of observed and estimated scores at 5, 10 and 15 over stages of a typical T20 innings. The symbols are the model-estimated scores at the respective stages coressponding to the observed scores on the horizontal axis. Ideally, the estimates must be equal to the observed scores and so should fall on the solid line. Panel 4: Bar–chart indicating observed and estimated runs scored in four equal over–groups in a typical T20 innings.

First innings score	10–over Kleiber target	10–over Kleiber par-score (0 wkts)	10–over DLS target	Kleiber Sum
80	49	32	48	81
120	72	49	72	121
160	96	65	94	161
200	120	81	114	201
240†	138	103	133	241

Table 1: 10–over Kleiber target and par–scores for different first innings totals. † See also Scenario 3.

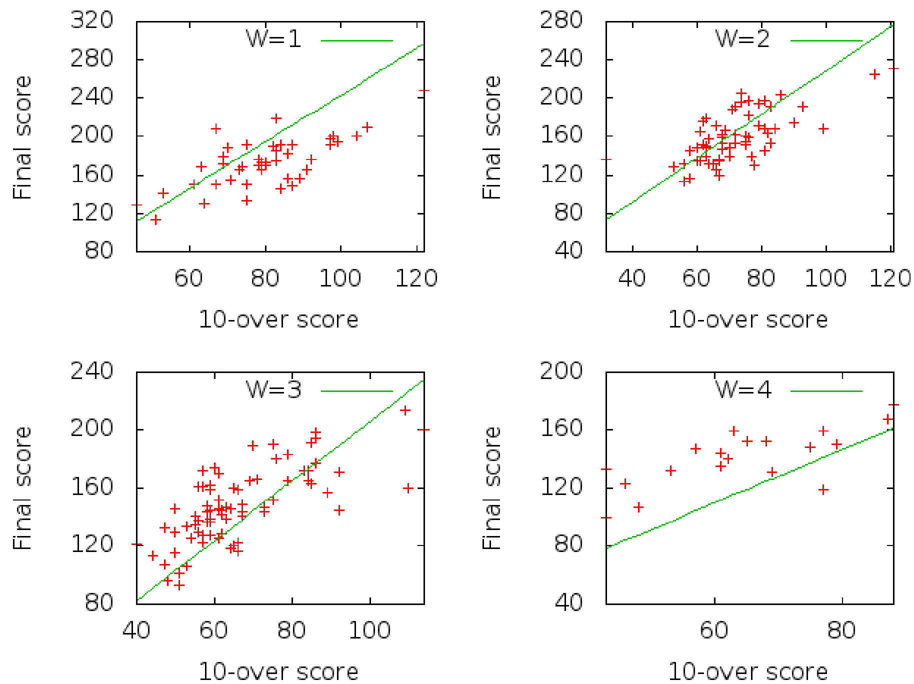

Figure 2: Comparison of observed and estimated final scores based on the 10–over scores and categorized by wickets lost. The symbols are the observed actual final scores and the solid line gives the estimates as modelled by the Kleiber rain rule

Match situation	10–over score in first innings	10–over target		5–over target	
		D/L	Kleiber	D/L	Kleiber
G1: First innings terminated due to rain after 10 overs	50 (for loss of 2 wkts.)	71	69	36	41
G2: Match started as 10–over game	68	–	69	–	41
G3: Match started as 10–over game	70	71	–	39	–

Table 2: Consistency for "equivalent" scenarios in different T20 games. The example is taken from vjdcricket.yolasite.com

The games G1 and G2 shown in Table 2 are "equivalent" for the Kleiber rain–rule because their 10–over targets are the same despite the nature of the 10–over first innings. On the same lines, games G1 and G3 are "equivalent" for the D/L. Consequently, we would expect the 5–over Kleiber targets for G1 and G2 as well as the 5–over D/L targets for G1 and G3 to be identical. The 5–over Kleiber targets are indeed same for both games, but the D/L targets differ by 3 runs, which is an inconsistency in the D/L. This inconsistency is likely carried over to DLS as well while the Kleiber rain rule and VJD do not suffer from this problem.

Scenario 3: Very high totals and "ultra"–short games

The strongest critique of D/L in T20, arguably, is that it favours the team batting second by setting low targets in the event of high first innings totals and very short second innings. This scenario is very likely in the shortest form of the game, and it needs to be seen how the Kleiber rule responds to such situations. Table 3 shows six different games pertaining to this scenario, with the Kleiber target being found in two different ways: one with a static exponent of $\frac{3}{4}$ and the other with a projected-score dependent exponent. In the latter case, the projected score PS is first calculated with $n = \frac{3}{4}$ and then the "modified" exponent is found as $n = \max(\frac{3}{4}, \tanh(\frac{PS}{215}))$. The resource percentages are then re-computed using this value of n,

$$B = \Big[\Big(\frac{O}{20}\Big)\Big(1 - \exp\Big(\frac{W-10}{W+1}\Big)\Big)\Big]^n$$

and so are the projected score and the Kleiber target. We skip the complete explanation behind this construction, but remark that it is based on the guiding principle that "revised targets must tend to those obtained using

Match situation	Score in first innings	5–over Kleiber target	
		$n = \frac{3}{4}$	$n = \max(\frac{3}{4}, \tanh(\frac{PS}{215}))$
G1: First innings terminated after 8 overs due to rain	90 (without loss)	101	77
G2: First innings terminated after 8 overs due to rain	90 (with loss of 3 wkts.)	79	75
G3: First innings terminated after 9 overs due to rain	104 (without loss)	103	78
G4: First innings completed without interruptions	260	93	83
G5: First innings completed without interruptions	210	75	75
G6: First innings completed without interruptions	190	68	68

Table 3: Kleiber rain rule applied to high first innings scores and effect of the non–static exponent

the simple average runs-per-over rule as first innings totals (or projected totals) become higher". The effect of this non–static exponent is evident from games G1 and G3 in the table. Clearly, the static exponent sets an unfair target in both games, while the "modified" exponent gives reasonable targets. This effect is relatively less pronounced in G2 and G4, although the "modified" exponent gives a lesser target than that obtained with a three-quarter exponent. For games G5 and G6, the "modified" exponent is equal to the static value of $\frac{3}{4}$ and a little effort will show that the differences appear only for first innings totals (or projected totals) that exceed 210.

The comparison

The Kleiber rain rule is a parametric rain rule but distinguishes itself from DLS and VJD in several respects. Firstly, Kleiber rain rule is exclusively meant for T20s. Secondly, the rule is bio–inspired and does not explicitly exploit historical data in *construction* of the mathematical model unlike DLS and VJD. Thirdly and most importantly, the mathematical basis of the rule is simple to understand and the rule itself easy to employ. Unlike DLS and VJD, both of which require a computer, Kleiber targets may be computed using a scientific calculator for the old–timers or with a Spreadsheet calculator (also distributed as part of this article) on a hand–held device for the gadget–driven generation. Nevertheless, the true litmus test is a quantitative comparison of the Kleiber rain rule with the D/L or DLS for realistic scenarios that have been encountered in practice. Table 4 presents this comparison for a selected set of T20 games, including the highly criticised England vs West Indies WC T20s. These examples serve to show the ability of the Kleiber rule to handle all possible scenarios, including those with multiple interruptions, quite efficiently. While DLS and Kleiber targets tend to agree for some of the cases, they differ considerably in games with short second innings, where the DLS targets are a considerable improvement over the D/L targets and generally tend to lie between the D/L and Kleiber targets.

The verdict

The Kleiber rain rule is a do–it–yourself and specialised rule for T20 cricket, and one whose details behind the mathematical model are available on public domain. It uses a parametric model but exhibits consistency of targets under different plausible scenarios, with the targets being reasonable even for multiple interruptions. This rain rule also gives a close approximation to the DLS target for many cases, and in those where they differ, provides food for thought. Put simply, the Kleiber rain rule is a spectator–friendly target resetting method and the cricket viewer now does not have to wait for a "magical" number to pop out on his television screen. The mathematical model is surprisingly simple, requiring only knowledge of high–school mathematics and the rules of the game, making it possibly the go-to rain rule for the players, administrators and the public alike. T20 has ushered in several innovations to the cricketing world like the lap scoop, reverse paddle and Super Over; the Kleiber rain rule may well be the latest addition. This rain rule has proved to be a consistent and fair approach for determining revised targets for interrupted T20 games and interested readers are encouraged to trial it for international and domestic T20s. The rule is based on the conjecture that the typical T20 innings may be described quite well with a simple power–law and the use of ever–growing data on T20s will supply the much needed conclusive proof. The performance of the Kleiber rain rule holds promise and could mark the beginning of the quest for spectator–friendly rules for fair determination of targets in interrupted cricket matches. It is true that no mathematical system is 100% perfect when it comes to interrupted limited–over matches and fittingly, the question on if and when the Kleiber rain rule could replace the DLS for T20s is best left to the cricket–loving public. For now though, forget the rain and enjoy the game.

Match situation	D/L Target	DLS Target	Kleiber Target
ENG 161 in 20 overs, Rain at break WI to chase in 9 overs	80	87	89
ENG 191 in 20 overs WI 30/0 in 2.2 overs, Rain stops play Match rescheduled to 6 overs	60	66	71
DD 143 in 20 overs. Rain at break and SRH to chase in 15 overs SRH 8/0 in 1.1 overs and match reduced to 12 overs, SRH 17/1 in 2 overs and match reduced to 5 overs	115 in 15 97 in 12 43 in 5	117 in 15 98 in 12 44 in 5	116 in 15 98 in 12 48 in 5
Perth Scorchers 193 in 20 overs, Rain at break reduces Sydney Sixers chase to 5 overs	54	63	69
WI 22/2 in 5 overs, Rain reduces match to 18 overs WI 132 in 18 overs, NZ to chase in 18 overs NZ 117/4 in 15 overs, Play called off	133 in 18 106	133 in 18 106	133 in 18 107
Perth Scorchers 159/1 in 15.2 overs Match reduced to 18 overs, Scorchers 183/2 in 18 overs Rain at break and Melbourne Stars to chase in 13 overs	139	145	151
Durham 174 in 20 overs; Northamptonshire 47/5 in 8 overs Target in 12 overs Game called off at 8 overs	– –	131 89	132 98

Table 4: Comparison of D/L, DLS and Kleiber targets for select interrupted T20s

The author would like to thank Prof. Steven Stern for several useful discussions and insights as well as for providing the DLS targets shown in the comparative study. For more details on the methodology and its performance, readers are referred to the authors' work *"Bio-allometry inspired resource estimation in Twenty20 cricket", Proceedings of IMechE, Part P: Journal of Sports Engineering and Technology.* The work is dedicated to the authors' cricketing group during his student days, the memories of which provided the initial impetus for this work.